

Reconstruction of Scalar Field Dark Energy Models in Kaluza-Klein Universe

M. Sharif ^{*} and Abdul Jawad [†]

Department of Mathematics, University of the Punjab,
Quaid-e-Azam Campus, Lahore-54590, Pakistan.

Abstract

This paper is devoted to study the modified holographic dark energy model by taking its different aspects in the flat Kaluza-Klein universe. We construct the equation of state parameter which evolves the universe from quintessence region towards the vacuum. It is found that the modified holographic model exhibits instability against small perturbations in the early epoch of the universe but becomes stable in the later times. We also develop its correspondence with some scalar field dark energy models. It is interesting to mention here that all the results are consistent with the present observations.

Keywords: Kaluza-Klein cosmology; Modified holographic dark energy; Scalar field models.

PACS: 04.50.Cd; 95.36.+x.

1 Introduction

During the last decades, astrophysicists and astronomers have made remarkable predictions about the gap of more than 70 percent of the overall energy density in the universe. Based on recent observations [1, 2], it has been

^{*}msharif.math@pu.edu.pk

[†]jawadab181@yahoo.com

made consensus that dark energy (DE) fills this gap. It is the unknown force having large negative pressure and referred to be responsible for accelerated expansion of the universe. Many efforts have been made about its identity but its nature is still unknown. The most obvious candidate of DE is the cosmological constant (or vacuum energy) but it suffers two major problems [3]. It is an interesting as well as the most challenging problem to find the best fit model of DE.

There are different dynamical DE models out of which holographic dark energy (HDE) model is the most prominent. It is proposed in the scenario of quantum gravity on the basis of holographic principle [4]. In fact, the derivation of this model is based on the argument [5] that *the vacuum energy (or the quantum zero-point energy) of a system with size L should always remain less than the mass of a black hole with the same size due to the formation of black hole in quantum field theory*. In mathematical form, we have $\rho_\Lambda = \frac{3c^2}{8\pi GL^2}$, known as HDE density [6, 7]. Here the constant $3c^2$ is used for convenience, L represents the infrared (IR) cutoff and G is the gravitational constant.

The choice of IR cutoff in the HDE model is very crucial. Li [7] remarked that instead of Hubble or particle horizons, the future event horizon should be the IR cutoff which shows compatibility with the present evolution of the universe. Later on, it was pointed out [8] that this choice of IR cutoff suffers the causality problem and proposed the age of the universe as IR cutoff, called agegraphic DE model. Some other proposals have also been given for the choice of IR cutoff. Granda and Oliveros [9] proposed that IR cutoff should be the function of square of the Hubble parameter and its derivative. This type of IR cutoff is motivated from the Ricci scalar of the FRW universe [10].

The scalar field DE models also belong to the family of dynamical DE models which explain the DE phenomenon. A wide variety of these models exists in literature including quintessence, K-essence, tachyon, phantom, ghost condensates and dilaton [11, 12]. Also, the well-known theories such as the supersymmetric, string and M theories cannot describe potential of the scalar field independently. It would be interesting to reconstruct the potential of DE models so that the scalar fields may describe the cosmological behavior of the quantum gravity.

The modification in the gravitational part and enhancement of dimensions in the original general relativity is another way to handle the DE puzzle. In higher dimensional theories, Kaluza-Klein (KK) theory [13] has been used

extensively for the purpose of cosmological implications. This theory has been described into ways, i.e., compact and non-compact forms depending on its fifth dimension. In its compact form, fifth dimension is like a circle having very small radius while in non-compact form, it behaves as a vacuum of 4D geometry. Moreover, the HDE has also been derived in higher dimensions with the help of the N -dimensional mass of the Schwarzschild black hole [14] known as modified holographic dark energy (MHDE) model [15, 16]. Sharif et al. [17]-[19] have investigated the evolution as well as generalized second law of thermodynamics of MHDE with Hubble and future event horizons as IR cutoffs in the flat and non-flat KK universe models. In a recent paper [20], the scalar field models are constructed for HDE with Hubble horizon and Granda and Oliveros cutoff as IR cutoff in flat and non-flat universes.

Here we use the Granda and Oliveros [9] IR cutoff for MHDE in flat KK universe. We discuss the evolution, instability and scalar field DE models in this scenario. The paper is organized as follows. Section 2 contains discussion of the evolution and instability of MHDE in flat KK universe. In section 3, we reconstruct scalar field models of MHDE. The last section summarizes the results.

2 Modified Holographic Dark Energy

In this section, we make analysis of the equation of state (EoS) parameter and instability of MHDE in compact flat KK universe [21] whose metric is given by

$$ds^2 = -dt^2 + a^2(t)[dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) + d\psi^2], \quad (1)$$

where $a(t)$ indicates the cosmic scale factor. The corresponding field equations are

$$H^2 = \frac{1}{6}\rho_\Lambda, \quad (2)$$

$$\dot{H} + 2H^2 = -\frac{1}{3}p_\Lambda, \quad (3)$$

where H is the Hubble parameter, dot indicates differentiation with respect to time and $8\pi G = 1$ for the sake of simplicity. Also, p_Λ and ρ_Λ are the pressure and energy density due to DE respectively. In order to derive MHDE, we use

the formula of the mass of the Schwarzschild black hole in N dimensions

$$M = \frac{(N-1)A_{N-1}}{16\pi G_N} r_H^{N-2},$$

where $8\pi G_N \equiv M_*$ is Planck mass in higher dimensions, A_{N-1} is the area of N unit spheres and r_H represents the size of the black hole. For KK universe, we take $N = 4$ and use the formula of area, it follows that

$$M = \frac{3\pi^2 M_*^3}{2} r_H^5.$$

Here, we assume $r_H = L = (\mu H^2 + \lambda \dot{H})^{-\frac{1}{2}}$ (μ and λ are positive constants, this IR cutoff is proposed by Granda and Oliveros [9]) and M_* should be less than the Planck length. This shows that horizon scale does not make the mass of five dimensional black hole larger than compactification scale of KK universe (i.e., of the order of Planck length), while it reduces its mass. With the help of Cohen et al. [5] relation, we can derive MHDE in the following form [17]

$$\rho_\Lambda = 3\pi^2 L^2 = \frac{3\pi^2}{\mu H^2 + \lambda \dot{H}}, \quad (4)$$

here we take M_* to be unity.

The equation of continuity for the MHDE become

$$\dot{\rho}_\Lambda + 4H(\rho_\Lambda + p_\Lambda) = 0. \quad (5)$$

Equations (2) and (4) lead to

$$\frac{dH^4}{dx} + \frac{4\mu}{\lambda} H^4 = \frac{2\pi^2}{\lambda}, \quad (6)$$

where $x = \ln a$. Solving the above equation, we obtain

$$H^4 = \frac{\pi^2}{2\mu} + b e^{\frac{-4\mu}{\lambda} x}, \quad (7)$$

where b is an integration constant. The EoS parameter for MHDE can be obtained by using Eqs.(4), (5) and (7)

$$\omega_\Lambda = -1 + \frac{\mu^2 b e^{\frac{-4\mu}{\lambda} x}}{\lambda(\pi^2 + 2\mu b e^{\frac{-4\mu}{\lambda} x})}. \quad (8)$$

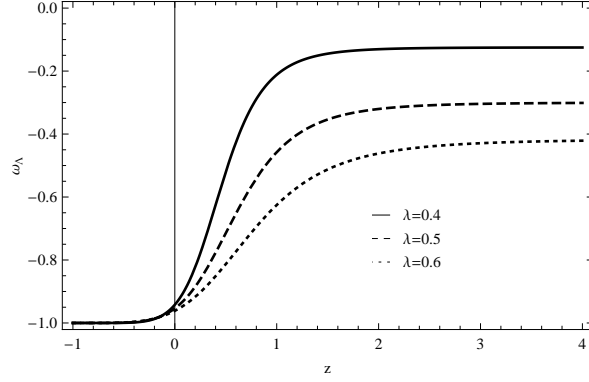


Figure 1: Plot of ω_Λ versus z for MHDE.

We plot ω_Λ versus redshift parameter z by using $a = a_0(1+z)^{-1}$ as shown in Figure 1 by taking $\mu = 0.7$, $b = 0.5$, $a_0 = 1$ and different values of $\lambda = 0.4, 0.5, 0.6$. It is observed that the EoS parameter evolves the universe from quintessence DE era towards vacuum DE era. This also shows that the parameter ω_Λ always remains in the quintessence region for $\lambda \geq 0.5$. However, it corresponds to the early inflation era for $\lambda < 0.5$ in the early time.

Now we explore the linear perturbation in order to examine the instability of MHDE. For this purpose, the square of the speed of sound (v_s^2) is evaluated which characterizes the stability of DE models. The speed of sound has the form [3]

$$v_s^2 = \frac{\dot{p}}{\dot{\rho}} = \frac{p'}{\rho'}, \quad (9)$$

where prime shows differentiation with respect to x . We would like to mention here that DE models are classically unstable as $v_s^2 < 0$ and vice versa. Equations (4), (5), (7) and (9) yield

$$v_s^2 = \frac{\pi^2(\mu - \lambda) - \mu b(2\lambda - \mu)e^{\frac{-4\mu}{\lambda}x}}{\lambda(\pi^2 + 2\mu b e^{\frac{-4\mu}{\lambda}x})}. \quad (10)$$

We plot the speed of sound versus z as shown in Figure 2 by keeping the same values of constant parameters as given above. This shows that the MHDE remains unstable in the early epoch and stable for the present and later time.

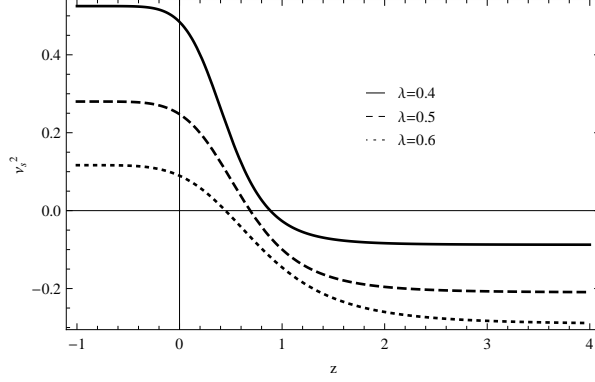


Figure 2: Plot of v_s^2 versus z for MHDE.

3 Correspondence of MHDE with Scalar Field Models

The current prediction of accelerated expansion of the universe has roots in the early inflationary epoch. It is argued that the inflation field has similar properties as the cosmological constant. Moreover, inspired by the inflation field, scalar field models have been proposed. Scalar field models are used in explaining the DE phenomenon due to their dynamical nature. These models describe the quintessence behavior of the universe and also provide the effective description of DE models through reconstruction scenario. In our case, the EoS for MHDE obeys this argument and hence it would be interesting to reconstruct the scalar field DE models in this scenario. We discuss all the results graphically with the same values of the constant parameters as given in the previous section.

3.1 Quintessence Dark Energy Model

This model is originated in the light of scalar field ϕ which is minimally coupled with gravity. The energy density and pressure of the quintessence DE model are given by [12]

$$\rho_q = \frac{1}{2}\dot{\phi}^2 + V(\phi), \quad p_q = \frac{1}{2}\dot{\phi}^2 - V(\phi), \quad (11)$$

where $\dot{\phi}^2$ is the kinetic energy and $V(\phi)$ is the potential of scalar field. This equation gives the EoS parameter as

$$\omega_q = \frac{\dot{\phi}^2 - 2V(\phi)}{\dot{\phi}^2 + 2V(\phi)}.$$

We identify $\rho_q = \rho_\Lambda$ and $p_q = p_\Lambda$ to establish the correspondence between MHDE and quintessence scalar field. Thus, it follows from Eq.(11) that

$$\dot{\phi}^2 = \frac{\mu\sqrt{6b}e^{\frac{-2\mu}{\lambda}x}}{\sqrt{\lambda(\pi^2 + 2\mu b e^{\frac{-4\mu}{\lambda}x})}}, \quad (12)$$

$$V(\phi) = \frac{6\pi^2\lambda - 3b\mu(\mu - 4\lambda)e^{\frac{-4\mu}{\lambda}x}}{\lambda\sqrt{2\mu(\pi^2 + 2\mu c e^{\frac{-4\mu}{\lambda}x})}}. \quad (13)$$

The value of scalar field turns out to be

$$\phi = \int_0^x \sqrt{\frac{6\mu^2 b e^{\frac{-4\mu}{\lambda}x}}{\lambda(\pi^2 + 2\mu b e^{\frac{-4\mu}{\lambda}x})}} dx. \quad (14)$$

The evolutionary trajectories of the scalar field ϕ and the corresponding scalar potential are shown in Figures 3 and 4. Here we use the initial condition of the scalar field $\phi(0) = 0$. We notice that the quintessence field increases initially but becomes flat at high redshift. This shows that the field decreases gradually with the expansion of the universe. Also, we observe that the quintessence potential becomes more steeper with the decrease of λ and tends to flat in the later epoch.

3.2 Tachyon Dark Energy Model

The energy and pressure of the tachyon DE model has the form [12]

$$\rho_t = \frac{V(\phi)}{\sqrt{1 - \dot{\phi}^2}}, \quad p_t = -V(\phi)\sqrt{1 - \dot{\phi}^2}. \quad (15)$$

The corresponding EoS parameter is

$$\omega_t = \dot{\phi}^2 - 1. \quad (16)$$

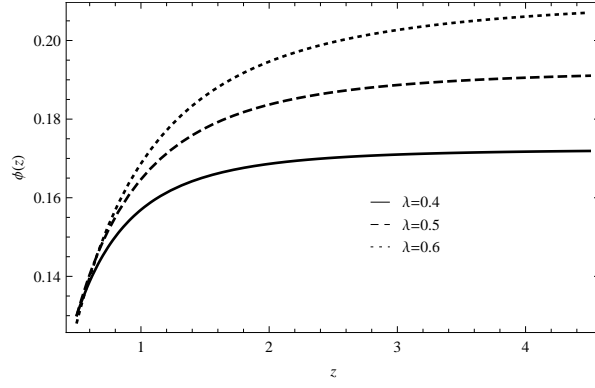


Figure 3: Plot of $\phi(z)$ versus z for quintessence model.

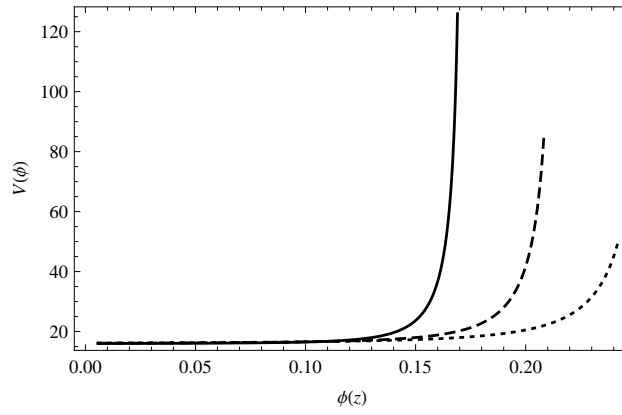


Figure 4: Plot of $V(\phi)$ versus $\phi(z)$ for quintessence model.

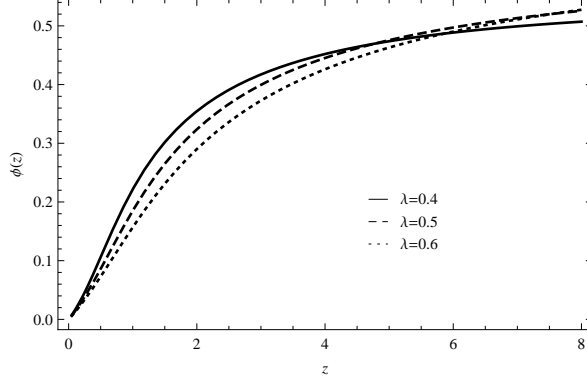


Figure 5: Plot of $\phi(z)$ versus z for tachyon model.

The correspondence between MHDE and tachyon model is obtained for $\rho_t = \rho_\Lambda$ and $p_t = p_\Lambda$, which leads to

$$\dot{\phi}^2 = \frac{\mu^2 b e^{\frac{-4\mu}{\lambda}x}}{\lambda(\pi^2 + 2\mu c e^{\frac{-4\mu}{\lambda}x})}, \quad (17)$$

$$V(\phi) = \sqrt{\frac{18(\pi^2\lambda + \mu b(2\lambda - \mu)e^{\frac{-4\mu}{\lambda}x})}{\mu\lambda}}. \quad (18)$$

Equations (7) and (17) give

$$\phi'(x) = \left(\frac{2\mu}{\pi^2 + 2\mu c e^{\frac{-4\mu}{\lambda}x}} \right)^{\frac{1}{4}} \sqrt{\frac{\mu^2 b e^{\frac{-4\mu}{\lambda}x}}{\lambda(\pi^2 + 2\mu b e^{\frac{-4\mu}{\lambda}x})}}. \quad (19)$$

We solve and plot it against z as shown in Figure 5. The evolution of tachyon field is very much similar to the quintessence field. The tachyon field attains the maximum value at early epoch and then decreases and goes towards zero at the present epoch. Also, we remark that the tachyon field rolls down potential slowly with the expansion of the universe as shown in Figure 6.

3.3 K-essence Dark Energy Model

The energy density and pressure of this model are [12]

$$\rho_k = V(\phi)(-\chi + 3\chi^2), \quad p_k = V(\phi)(-\chi + \chi^2), \quad (20)$$

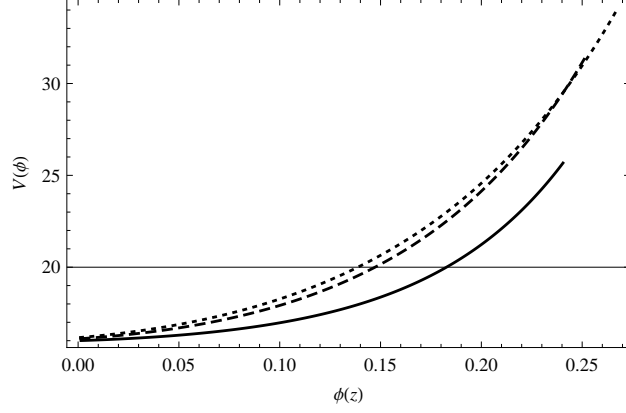


Figure 6: Plot of $V(\phi)$ versus $\phi(z)$ for tachyon model.

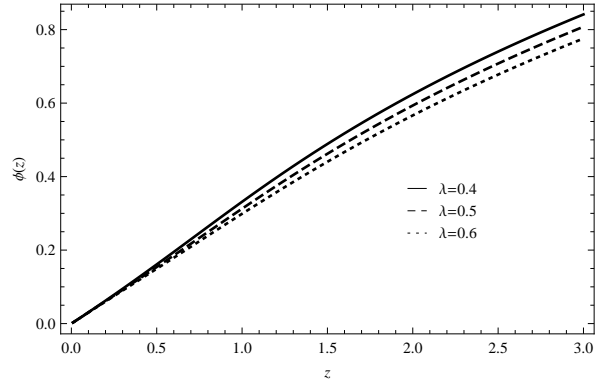


Figure 7: Plot of $\phi(z)$ versus z for K-essence model.

where $\chi = \frac{1}{2} \dot{\phi}^2$ and $V(\phi)$ represents the scalar potential of K-essence model. This leads to the following EoS parameter for tachyon DE model

$$\omega_k = \frac{1 - \chi}{1 - 3\chi}. \quad (21)$$

We set $\rho_k = \rho_\Lambda$ and $p_k = p_\Lambda$ for the correspondence between MHDE and

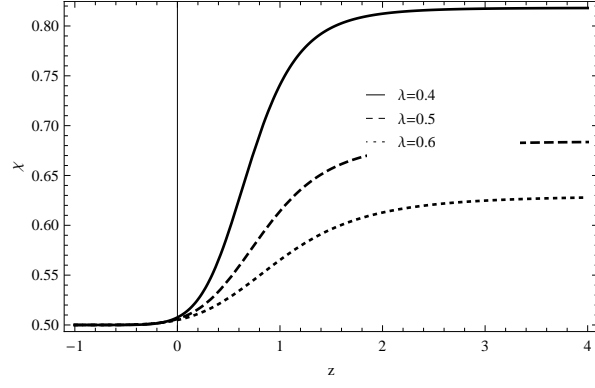


Figure 8: Plot of χ versus z for K-essence model.

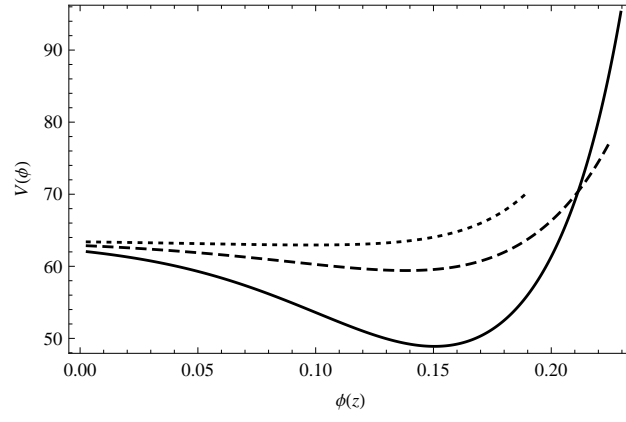


Figure 9: Plot of $V(\phi)$ versus $\phi(z)$ for K-essence model.

K-essence model and obtain

$$\chi = \frac{2\pi^2\lambda + \mu b(4\lambda - \mu)e^{\frac{-4\mu}{\lambda}x}}{4\pi^2\lambda + \mu b(8\lambda - 3\mu)e^{\frac{-4\mu}{\lambda}x}}, \quad (22)$$

$$V(\phi) = \frac{3(1 - 3\omega_\Lambda)^2}{1 - \omega_\Lambda} \left(\frac{2\mu}{\pi^2 + 2\mu ce^{\frac{-4\mu}{\lambda}x}} \right)^{\frac{1}{2}}. \quad (23)$$

Finally, the expression $\chi = \frac{1}{2}\dot{\phi}^2$ leads to

$$\phi'(x) = \left(\frac{2\mu}{\pi^2 + 2\mu ce^{\frac{-4\mu}{\lambda}x}} \right)^{\frac{1}{4}} \sqrt{\frac{2(2\pi^2\lambda + \mu b(4\lambda - \mu)e^{\frac{-4\mu}{\lambda}x})}{4\pi^2\lambda + \mu b(8\lambda - 3\mu)e^{\frac{-4\mu}{\lambda}x}}}. \quad (24)$$

Figure 7 shows that the K-essence scalar field decreases with the increment of MHDE parameter λ . The EoS of tachyon DE model is compatible with the accelerated expansion of the universe in the range $\frac{1}{3} < \omega_\Lambda < \frac{2}{3}$. It is noted that the kinetic term is consistent with this EoS for $\lambda = 0.5, 0.6$, but it is inconsistent for $\lambda \leq 0.4$ as shown in Figure 8. Also, the plot 9 indicates that the K-essence potential increases with the increase of the field ϕ . It rolls down the potential because K-essence scalar field decreases with the expansion of the universe.

3.4 Dilaton Dark Energy Model

The Lagrangian of dilaton field describes the pressure of scalar field given by [11]

$$p_d = -\chi + me^{n\phi}\chi^2, \quad (25)$$

where m and n are taken as positive constants. The corresponding energy density is

$$\rho_d = -\chi + 3me^{n\phi}\chi^2. \quad (26)$$

This model has the EoS parameter

$$\omega_d = \frac{-1 + me^{n\phi}\chi}{-1 + 3me^{n\phi}\chi}. \quad (27)$$

The replacement of ρ_d by ρ_Λ and p_d by p_Λ (for correspondence) gives

$$e^{n\phi}\chi = \frac{2\pi^2\lambda + \mu b(4\lambda - \mu)e^{\frac{-4\mu}{\lambda}x}}{m(4\pi^2\lambda + \mu b(8\lambda - 3\mu)e^{\frac{-4\mu}{\lambda}x})}. \quad (28)$$

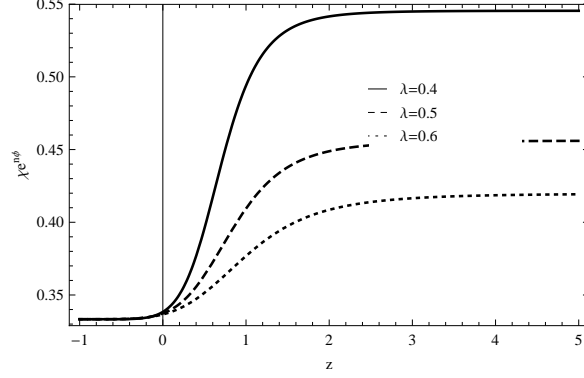


Figure 10: Plot of $e^{n\phi}\chi$ versus z for dilaton field.

Its plot against z with $m = 1.5$ and $n = 0.05$ is shown in Figure 10. We observe that $e^{n\phi}\chi$ shows consistency with the expanding scenario of the universe predicated by EoS of this model. The solution of the above equation follows

$$\begin{aligned} \phi(x) = & \frac{2}{n} \ln \left[1 + \frac{n}{\sqrt{2m}} \int_0^x \sqrt{\frac{2\pi^2\lambda + \mu b(4\lambda - \mu)e^{\frac{-4\mu}{\lambda}x}}{4\pi^2\lambda + \mu b(8\lambda - 3\mu)e^{\frac{-4\mu}{\lambda}x}}} \right. \\ & \times \left. \left(\frac{2\mu}{\pi^2 + 2\mu b e^{\frac{-4\mu}{\lambda}x}} \right)^{\frac{1}{4}} dx \right]. \end{aligned} \quad (29)$$

Its graphical presentation is shown in Figure 11 which exhibits direct proportionality with respect to z leading to the scaling solutions for dilaton model [11].

4 Summary

It is believed that our universe expands with accelerated expansion and the scalar field DE models act as an effective theories of an underlying phenomenon of DE. We investigate the evolution of MHDE, instability and reconstructed scalar field DE models. We consider MHDE with IR cutoff as a function of the Hubble parameter and its derivative in the flat Kaluza-Klein universe. This type of DE density avoids the causality problem due to dependence on the local quantities. The MHDE parameter λ plays the crucial

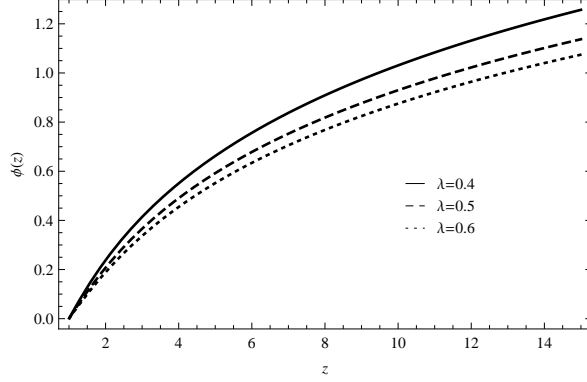


Figure 11: Plot of $\phi(z)$ versus z for dilaton field.

role in evaluating the results. We have used the best fit values of $\mu = 0.7$ and $\lambda = 0.4, 0.5, 0.6$ obtained by Wang and Xu [22] through type Ia supernovae, baryon acoustic oscillations, CMBR and the observational Hubble data. The results are summarized as follows.

- The EoS parameter ω_Λ shows transition from quintessence DE era ($-\frac{1}{3} < \omega < -1$) towards vacuum DE era ($\omega = -1$) as displayed in Figure 1. This type of behavior has led to reconstruct the scalar field DE models. Also, our result of the present value of $\omega_\Lambda = -0.95$ at $z = 0$ (for $\lambda = 0.4, 0.5, 0.6$) is almost closer to the value obtained for HDE in GR [7, 9, 23, 24].
- It is interesting to check the viability of the MHDE model due to its dependence upon local quantities. Using the squared speed of sound, we have found that the MHDE with new IR cutoff is unstable for early epoch but stable for the later time as shown in Figure 2. It is mentioned here that the MHDE is always stable for $\lambda < 0.35$, while in GR, the new HDE is unstable [25]. In addition, the Chaplygin and tachyon Chaplygin gases are stable as $v_s^2 > 0$, but the holographic [26], agegraphic [27] and QCD ghost DE [28] models are classically unstable as $v_s^2 < 0$.
- Finally, we have provided the correspondence of MHDE with scalar field DE models including quintessence, tachyon, K-essence and dilaton models. In all these models, the scalar field shows the decreasing behavior with the expansion of the universe. The scalar potential increases

and becomes steeper with the increase of scalar field ϕ for quintessence, tachyon and K-essence DE models. The plots of scalar field DE models are given in Figures **3–11**. In K-essence and dilaton DE models, the kinetic terms exactly lie in the required region (where EoS parameter predicts the accelerated expansion of the universe). We would like to remark here that the results of scalar field and corresponding potential are consistent with the current status of the universe. These are also consistent with the results of Zhang [23] for quintessence HDE, for tachyon HDE [24, 29, 30] and Rozas-Fernández [31] for dilaton HDE models.

References

- [1] Bennett, C. L. et al.: *Astrophys. J.* **148**(2003)1.
- [2] Tegmark, M. et al.: *Phys. Rev. D* **69**(2004)103501.
- [3] Peebles, P.J.E.: *Rev. Mod. Phys.* **75**(2003)559.
- [4] Susskind, L.: *J. Math. Phys.* **36**(1995)6377.
- [5] Cohen, A., Kaplan, D. and Nelson, A.: *Phys. Rev. Lett.* **82**(1999)4971.
- [6] Hsu, S.D.H.: *Phys. Lett. B* **594**(2004)13.
- [7] Li, M.: *Phys. Lett. B* **603**(2004)1.
- [8] Wei, H. and Cai, R.G.: *Phys. Lett. B* **660**(2008)113.
- [9] Granda, L. and Oliveros, A.: *Phys. Lett. B* **669**(2008)275.
- [10] Gao, C., Chen, X. and Shen, Y.G.: *Phys. Rev. D* **79**(2009)043511.
- [11] Piazza, F. and Tsujikawa, S.: *JCAP* **07**(2004)004.
- [12] Copeland, E.J., Sami, M. and Tsujikawa, S.: *Int. J. Mod. Phys. D* **15**(2006)1753.
- [13] Kaluza, T.: *Zum Unitatsproblem der Physik* Sitz. Press. Akad. Wiss. Phys. Math. kl (1921)966; Klein, O.: *Zeits. Phys.* **37**(1926)895.

- [14] Myers, R.C.: Phys. Rev. D **35**(1987)455.
- [15] Gong, Y. and Li, T.: Phys. Lett. B **683**(2010)241.
- [16] Liu, D.J., Wang, H. and Yang, B.: Phys. Lett. B **694**(2010)6.
- [17] Sharif, M. and Khanum, F.: Gen. Relativ. Gravit. **43**(2011)2885.
- [18] Sharif, M. and Jawad, A.: Astrophys. Space Sci. **337**(2012)789.
- [19] Sharif, M. and Jawad, A.: Eur. Phys. J. C **72**(2012)1901.
- [20] Granda, L. and Oliveros, A.: Phys. Lett. B **671**(2009)199; Karami, K. and Fehri, J.: Phys. Lett. B **684**(2010)61; Sheykhi, A.: Phys. Rev. D **84**(2011)107302.
- [21] Ozel, C., Kayhan, H. and Khadekar, G.S.: Ad. Studies. Theor. Phys. **4**(2010)117.
- [22] Wang, Y and Xu, L.: Phys. Rev. D **81**(2010)083523.
- [23] Zhang, X.: Phys. Lett. B **648**(2007)1.
- [24] Setare, M.R.: Phys. Lett. B **653**(2007)116.
- [25] Malekjani, M and Khodam-Mohammadi, A.: Int. J. Mod. Phys. D **20**(2011)281.
- [26] Myung, Y.S.: Phys. Lett. B **652**(2007)223.
- [27] Kim, K.Y., Lee, H.W. and Myung, Y.S.: Phys. Lett. B **660**(2008)118.
- [28] Ebrahimi, E. and Sheykhi, A.: Int. J. Mod. Phys. D **20**(2011)2369.
- [29] Zhang, J., Zhang, X. and Liu, H.: Phys. Lett. B **651**(2007)84.
- [30] Rozas-Fernández, A., Brizuela, D. and Cruz, N.: Int. J. Mod. Phys. D **19**(2010)573.
- [31] Rozas-Fernández, A.: Eur. Phys. J. C **71**(2011)1536.